An Approach to Tracking a 3D-Target with 2D-Radar

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SUMMARY & CONCLUSION

Tracking 3D-targets with a 2D-radar is a significant and challenging problem. Motivated by the Range-Parameterized EKF (RPEKF) algorithm used in the problem of bearing-only tracking, we present a practical solution to solve this problem in this article, i.e., Height-Parameterized EKF (HPEKF). This method could reduce the fuzzy phenomenon in height information of the target. So we can track 3D-targets with a single 2D-radar in an effective way under some conditions.

1. INTRODUCTION

It is well known that we can only get target bearing and target range from a 2D-radar. To eliminate the fuzzy phenomenon when using 2D-radar to track 3D- targets, the geometric method is generally adopted. However, it is indicated theoretically and experimentally that this method has poor performance especially when the range is far or the altitude is low [1-4]. Then, Tracking 3D-targets using such insufficient information is a significant and challenging problem.

Although there exist some similarities, the problem of tracking a 3D-target with a 2D-radar has apparent difference from the problem of bearing-only tracking. In both of the tracking problems, the information gained by the observation platform is incomplete. However, because we can get target range from a 2D-radar, the target in the former situation is observable if the height of the target is invariable. But the performance of tracking is highly dependent on the accuracy of initial height, and in general, the height is not known beforehand. To solve this problem, we present a practical algorithm in this article, i.e., Height-Parameterized EKF (HPEKF). This method is activated by the Range-Parameterized EKF (RPEKF) [5, 6] used in the bearing-only tracking problem. Our main ideas is that, to reduce the fuzzy phenomenon in the height information, we divide the height interval of interest into a number of subintervals, and each subinterval is dealt with an independent EKF. The weights associated with each EKF are computed to determine how the state estimate of each filter is combined. And the updated state estimate and covariance matrix of the height-parameterized tracker can be computed as a weighted sum of the individual estimates and covariance matrices.

2. PROBLEM STATEMENT

Let us consider a 3D-target, located at coordinates $(x(t), y(t), z(t))$ moves with a nearly constant velocity vector $(\dot{x}(t), \dot{y}(t), \dot{z}(t))$ and nearly constant height. Then the target is defined to have the state vector

$$\mathbf{x}(t) = [x(t), y(t), z(t), \dot{x}(t), \dot{y}(t), \dot{z}(t)]',$$

where the prime denotes transpose, and the discrete time state equation for this problem can be written as

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14. ABSTRACT

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$$\mathbf{x}_{k+1} = \Phi_{k+1|k}\mathbf{x}_k + \Gamma_k\mathbf{w}_k , \tag{1}$$

where

$$\Phi_{k+1|k} = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} ,$$

$$\Gamma_k = \begin{bmatrix} T^2/2 & 0 & 0 \\ 0 & T^2/2 & 0 \\ 0 & 0 & T^2/2 \\ T & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{bmatrix} ,$$

T is the sampling time, $\mathbf{w}_k$ is an i.i.d process noise vector with $\mathbf{w}_k \sim N(\mathbf{0}, Q_0)$. Where $Q_0 = \sigma_a^2 I_3$, σ_a is a scalar, I_3 is the 3×3 identity matrix.

The available measurement at time k encompasses the target bearing $\theta(k)$ and target range $r(k)$. It can be modeled as

$$\mathbf{z}_k = H(\mathbf{x}_k) + \mathbf{v}_k = \breve{\mathbf{z}}_k + \mathbf{v}_k , \tag{2}$$

where $\mathbf{v}_k$ is an independent Gaussian measurement noise vector with $\mathbf{v}_k \sim N(\mathbf{0}, R_0)$, Here $R_0 = diag(\sigma_r^2, \sigma_\theta^2)$, whereas $\breve{\mathbf{z}}_k$ is the noise-free bearing and range vector

$$\breve{\mathbf{z}}_k = H(\mathbf{x}_k) = \begin{bmatrix} \breve{r}(k) \\ \breve{\theta}(k) \end{bmatrix} = \begin{bmatrix} \sqrt{x^2(k)+y^2(k)+z^2(k)} \\ \tan^{-1}\left(\dfrac{x(k)}{y(k)}\right) \end{bmatrix} . \tag{3}$$

Given a sequence of measurements $\mathbf{z}_k$, $k = 1,2,\cdots$, defined by (2) and (3), and target motion model described in (1), the tracking problem is to obtain estimate of the state vector $\mathbf{x}_k$.

3. HEIGHT-PARAMETERIZED EKF

The HPEKF tracks the state of a 3D-target with a number of independent EKF trackers, each with a different initial height estimate . To do so, the height interval of interest is divided into a number of subintervals, and each subinterval is dealt with an independent EKF. Suppose the height interval of interest is $(z_{\min}, z_{\max})$, and we wish to track using N_F EKF filters. For a particular EKF, we note that the tracking performance is highly dependent on the *Coefficient of Variation* the height estimate [5], C_z, given by σ_z/z, where z and σ_z are the height estimate and its standard deviation respectively. In order to maintain a comparable performance for all N_F filters, it is desirable to subdivide the interval $(z_{\min}, z_{\max})$ such that C_z is the same for each subinterval. Note that C_z for each subinterval may be computed approximately as σ_{z_i}/z_i, where z_i is the mean of subinterval i and σ_{z_i} is the height standard deviation for that subinterval.

Assuming the height errors to be uniformly distributed in each subinterval, the desirable subdivision can be obtained if the subinterval boundaries are chosen as a geometrical progression. If ρ is the common ratio, we have the relation

$$z_{\max} = z_{\min}\rho^{N_F} ,$$

which gives ρ as

$$\rho = \left(\frac{z_{\max}}{z_{\min}}\right)^{1/N_F} .$$

For the above division of range, it is easily established [5] that the coefficient of variation is given by

$$C_z = \frac{\sigma_{z_i}}{z_i} = \frac{2(\rho-1)}{\sqrt{12}(\rho+1)} . \tag{4}$$

To determine how the state estimate of each filter is combined, we need to compute the weights associated with each EKF. At time step 1, let the probability that the true track originated from the i-th subinterval be denoted by w_1^i.

These probabilities of the distribution of target height, which form the initial weights for the HP tracker, is supposed known, e.g., Gaussian distribution. If no prior information about the true height is available, we get the probabilities from a

uniform distribution. The corresponding probabilities at time k can be computed recursively according to Bayes' rule,

$$w_k^i = \frac{p(\mathbf{z}_k \mid i)w_{k-1}^i}{\sum_{j=1}^{N_F} p(\mathbf{z}_k \mid j)w_{k-1}^j}, \qquad (5)$$

where $p(\mathbf{z}_k \mid i)$ is the likelihood of measurement $\mathbf{z}_k$, given that the target originated in subinterval i. Assuming Gaussian statistics, this can be computed as

$$p(\mathbf{z}_k \mid i) = \frac{1}{2\pi|R_i|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2}\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}^i\right)' R_i^{-1}\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}^i\right)\right\}, \quad (6)$$

where $\hat{\mathbf{z}}_{k|k-1}^i = \left(\hat{r}_{k|k-1}^i, \hat{\theta}_{k|k-1}^i\right)'$ is the predicted measurement at k for filter i, and R_i is the innovation variance for filter i given by

$$R_i = H_k^i P_{k|k-1}^i H_k^{i'} + R_0, \qquad (7)$$

where

$$H_k^i = \left.\frac{\partial H(\mathbf{x})}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k|k-1}^i}$$

$$= \left[\begin{array}{cccccc} \dfrac{x}{\sqrt{x^2+y^2+z^2}} & \dfrac{y}{\sqrt{x^2+y^2+z^2}} & \dfrac{z}{\sqrt{x^2+y^2+z^2}} & 0 & 0 & 0 \\ \dfrac{y}{x^2+y^2} & -\dfrac{x}{x^2+y^2} & 0 & 0 & 0 & 0 \end{array}\right]_{\hat{\mathbf{x}}_{k|k-1}^i}$$

is the linearised measurement matrix, and $P_{k|k-1}^i$ is the predicted covariance for filter i, and R_0 is the measurement variance matrix.

Now, suppose that the updated state estimate of filter i is denoted by $\hat{\mathbf{x}}_{k|k}^i$. Then, the updated state estimate of the height-parameterized tracker can be computed as a weighted sum of the individual estimates,

$$\hat{\mathbf{x}}_{k|k} = \sum_{i=1}^{N_F} w_k^i \hat{\mathbf{x}}_{k|k}^i. \qquad (8)$$

Similarly, if $P_{k|k}^i$ denotes the covariance matrix of the i-th filter at k, the corresponding covariance for the height-parameterized tracker may be computed as

$$P_{k|k} = \sum_{i=1}^{N_F} w_k^i \left[P_{k|k}^i + \left(\hat{\mathbf{x}}_{k|k}^i - \hat{\mathbf{x}}_{k|k}\right)\left(\hat{\mathbf{x}}_{k|k}^i - \hat{\mathbf{x}}_{k|k}\right)'\right]. \qquad (9)$$

The tracker initialization for HPEKF is carried out according to standard initialization techniques for single EKF trackers. In particular, the state vector and its covariance are initialized as [8]

$$\hat{\mathbf{x}}_{0|0}^i = \left(\sqrt{r_1^2 - z_i^2}\sin\theta_1, \sqrt{r_1^2 - z_i^2}\cos\theta_1, z_i, 0, 0, 0\right)',$$

$$P_{0|0}^i = diag\left(\sigma_{z_i}^2, \sigma_{z_i}^2, \sigma_{z_i}^2, \sigma_v^2, \sigma_v^2, \sigma_v^2\right),$$

where $\mathbf{z}_1 = (r_1, \theta_1)$ is the initial 2D-radar report, z_i and σ_{z_i} are mean and standard deviation of height estimate for interval i assuming uniform distribution of errors, and σ_v is the velocity standard deviation.

The HP tracker has to implement N_F EKFs if all the height subintervals are processed throughout. However, it has been found that in a majority of target-observer scenarios, the weighting of some of the subintervals rapidly reduce to zero. In such cases, the corresponding filters can be removed from the tracking process without loss of accuracy, thereby reducing the processing requirement. Thus, a weighting threshold can be set and any filter corresponding to a subinterval with a weight less than the threshold may be removed from the tracking process.

4. ALGORITHM FOR HPEKF

Based on the elements presented above, it is possible to propose the following generic algorithm for HPEKF.

Algorithm for HPEKF

1. Divide the height interval into N_F subintervals.

2. Initialization: at $k = 1$, for $i = 1, 2, \cdots, N_F$, set

$$\hat{\mathbf{x}}_{0|0}^i = \left(\sqrt{r_1^2 - z_i^2}\sin\theta_1, \sqrt{r_1^2 - z_i^2}\cos\theta_1, z_i, 0, 0, 0\right)',$$

$$P_{0|0}^i = diag\left(\sigma_{z_i}^2, \sigma_{z_i}^2, \sigma_{z_i}^2, \sigma_v^2, \sigma_v^2, \sigma_v^2\right).$$

3. At time k, for $i = 1, 2, \cdots, N_F$

- Implement EKFs parallel:

$$\hat{\mathbf{x}}^i_{k|k-1} = \Phi_{k|k-1}\hat{\mathbf{x}}^i_{k-1|k-1} + \Gamma_{k-1}\mathbf{w}_{k-1};$$

$$P^i_{k|k-1} = \Phi_{k|k-1}P^i_{k|k}\Phi'_{k|k-1};$$

$$K^i_k = P^i_{k|k-1}H^{i'}_k\left(H^i_k P^i_{k|k-1}H^{i'}_k + R_0\right)^{-1};$$

$$\hat{\mathbf{z}}^i_{k|k-1} = \left(\hat{r}^i_{k|k-1},\hat{\theta}^i_{k|k-1}\right)' = H\left(\hat{\mathbf{x}}_{k|k-1}\right);$$

$$\hat{\mathbf{x}}^i_{k|k} = \hat{\mathbf{x}}^i_{k|k-1} + K^i_k\left(\mathbf{z}_k - \hat{\mathbf{z}}^i_{k|k-1}\right);$$

$$P^i_{k|k} = \left(I - K^i_k H^i_k\right)P^i_{k|k-1}.$$

- Compute the weights associated with each EKF:

$$w^i_k = \frac{p(\mathbf{z}_k \mid i)w^i_{k-1}}{\sum_{j=1}^{N_F} p(\mathbf{z}_k \mid j)w^j_{k-1}};$$

where

$$p(\mathbf{z}_k \mid i)$$
$$= \frac{1}{2\pi|R_i|^{1/2}}\exp\left\{-\frac{1}{2}\left(\mathbf{z}_k - \hat{\mathbf{z}}^i_{k|k-1}\right)'R_i^{-1}\left(\mathbf{z}_k - \hat{\mathbf{z}}^i_{k|k-1}\right)\right\}$$

- Compute the state estimate and the corresponding covariance of the target:

$$\hat{\mathbf{x}}_{k|k} = \sum_{i=1}^{N_F} w^i_k \hat{\mathbf{x}}^i_{k|k};$$

$$P_{k|k} = \sum_{i=1}^{N_F} w^i_k\left[P^i_{k|k} + \left(\hat{\mathbf{x}}^i_{k|k} - \hat{\mathbf{x}}_{k|k}\right)\left(\hat{\mathbf{x}}^i_{k|k} - \hat{\mathbf{x}}_{k|k}\right)'\right].$$

5. *SIMULATION RESULTS*

The example considers a target that starts at $x = 1000$ m, $y = 50000$ m and $z = 5000$ m and travels along with the x axis at a speed of $300 m/s$. For this illustrative example, a rotating radar with a scan period of 1s provides range and bearing measurement. The radar provides range measurements with a standard deviation of 150m and bearing measurement with a standard deviation of $0.7°$. The range and bearing measurements are processed via extended Kalman filter equations. The results are an average of Monte Carlo simulations with 100 trials.

Figure 1 shows the RMSEs in the height estimates for the EKF with 5000m initial height. Figure 2 and figure 3 show the RMSEs in the height estimates for EKF with 4000m and 7000m initial height respectively. It can be find that the height estimates sequence is convergent and the estimate errors are small if the initial height is relative accurate. If the initial height is not accurate, the height estimates sequence is divergent. Figure 4 shows the comparison of the RMSEs in the height estimates for the EKFs with initial velocities along with the x axis $300 m/s$, $400 m/s$ and $500 m/s$ respectively. Note that the influence of the velocity initialization is very small. Figure 5 shows the RMSEs in the height estimates for the HPEKF. From this figure we can see that the height estimates sequence is convergent, and the height estimate errors are close to the height estimate errors for the EKF with real height of the target as its initial height.
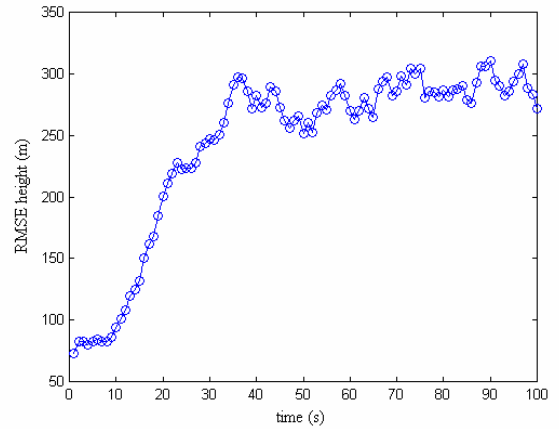
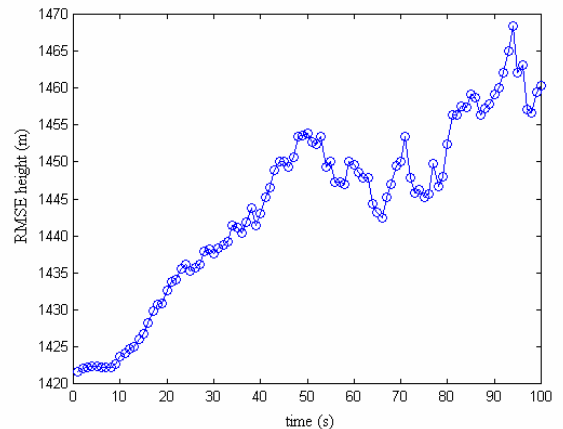

Fig. 1 RMS height error with 5000m initial height

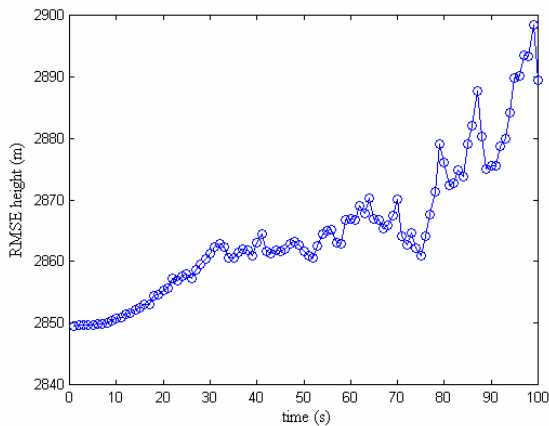Fig. 2 RMS height error with 4000m initial height

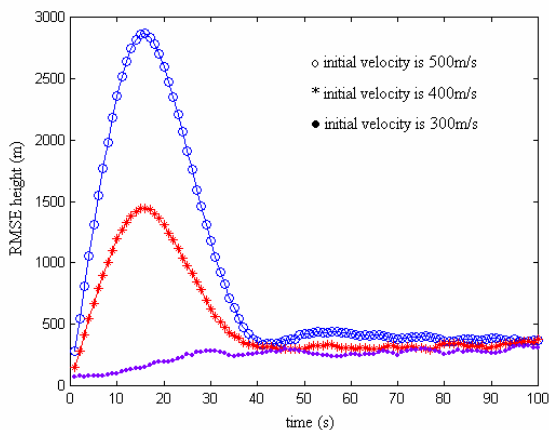Fig. 3 RMS height error with 7000m initial height

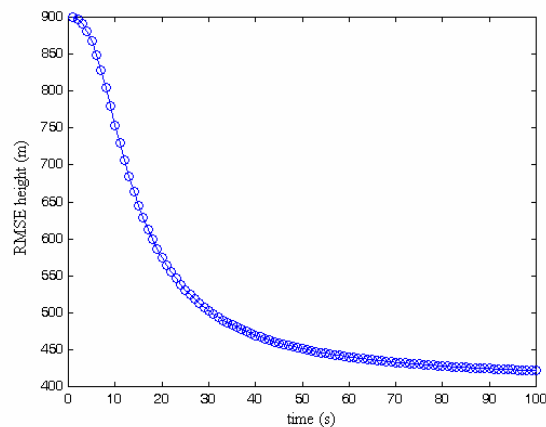Fig. 4 RMS height error comparison with different velocity
initializations

Fig. 5 RMS height error by the HPEKF proposed in this paper

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